B.Sc. Hons. Part III 7th Paper

Differential Equations

Alok law
0617

## Standard forms II

Form : $\boxed{z = px + qy + f(p,q)}$ —(1)

**Complete soln**

put $p = a$ and $q = b$

$$\therefore \underbrace{z = ax + by + f(a,b)}_{\text{complete soln}} \quad \text{——(2)}$$

**General integral**

put $b = \phi(a)$ in (2)

$$\Rightarrow z = ax + y\phi(a) + f\{a, \phi(a)\} \quad \text{——(3)}$$

Differentiating it with respect to a, we get

$$0 = x + y\,\phi'(a) + f'\{a, \phi(a)\} \quad \text{——(4)}$$

Eliminating a from (3) and (4) gives the general integral.

**Singular soln (integral)**

Diff. (2) partially w.r.to a, we get

$$0 = x + \frac{\partial f}{\partial a} \quad \text{——(5)}$$

Diff (2) partially w.r.to b, we get

$$0 = y + \frac{\partial f}{\partial b} \quad \text{——(6)}$$

Elimination of a and b from (2), (5), (6) gives the singular integral.

**Q.** Solve $z = px + qy + pq$.

**Soln** The given eqn.

$$z = px + qy + pq \quad \text{---(1)}$$

It is of the form $z = px + qy + f(p,q)$

$\therefore$ Its complete integral is given by

$$z = ax + by + ab \quad \text{---(2)}$$

[i.e by putting $p = a$ and $q = b$ in (1)]

Put $b = \phi(a)$

$\therefore (2) \Rightarrow \quad z = ax + y\phi(a) + a\phi(a) \quad \text{---(3)}$

Diff. it partially w.r. to $a$,

$$0 = x + y\phi'(a) + \phi(a) + a\phi'(a) \quad \text{---(4)}$$

By eliminating $a$ from (3) and (4) gives the general integral.

Diff. (2) partially w.r. to $a$, we get

$$0 = x + b \Rightarrow x = -b \quad \text{---(5)}$$

Diff (2) partially w.r. to $b$, we have

$$0 = 0 + y + a \Rightarrow y = -a \quad \text{---(6)}$$

So (2) becomes

$$z = a \times (-b) + b \times (-a) + ab$$

$$\Rightarrow z = ab = xy$$

$\therefore \boxed{z = xy}$ is the singular integral